Gas-Solid and Gas-Liquid Mass-Transfer Coefficients

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In 1934 Gilliland and Sherwood presented the data on the rates of vaporization of nine liquids into air in a wetted-wall column, in order to test the analogy of Colburn (1930). The results obtained were interpreted as gas-solid transport and correlated by:

$$Sh = 0.023\, Re^{0.83} Sc^{0.44} \tag{1}$$

The exponent on the Schmidt number differed from 0.33 expected from the Colburn correlation. Equation 1 is often cited as a correlation for the tube gas-solid mass-transfer coefficients (Sherwood and Pigford, 1952; Foust et al., 1962; Coulson and Richardson, 1965; Skelland, 1974; Welty et al., 1976; Treybal, 1980). However, some of the texts also offer an alternative relation:

$$Sh = 0.023 Re^{0.83} Sc^{0.33} \tag{2}$$

as a general equation for turbulent mass transfer in pipes for both gases and liquids. The discrepancy in the Schmidt number exponent is usually explained by the small range of the Schmidt number ($0.06 \leq Sc \leq 2.5$) covered by the experiments of Gilliland and Sherwood (1934).

The scope of this article is to elucidate and provide evidence that the proper equation for turbulent mass-transfer coefficients in tubes for both gases and liquids is Eq. 2. *The different Schmidt number exponent in Eqs.* 1 *and* 2 *is not a consequence of experimental uncertainty, but results from the fact that in wetted-wall columns we have gas-liquid, not gas-solid mass transfer.* Namely, we assert that Gilliland and Sherwood (1934) and Barnet and Kobe (1941), and for that matter all others that used wetted wall columns, measured gas-liquid mass transfer, that is, transfer between the turbulent gas core and a thin liquid film flowing at the wall. According to Levich (1959) near the fluid-fluid interface eddy viscosity varies as a second power of distance from the wall:

$$\epsilon = \gamma_3 y^2 \tag{3}$$

leading to a power 0.5 (on Sc-number) in the $Sh = f(Re,Sc)$ relation. This is different from the fluid-solid systems where eddy viscosity varies with a third power with distance from the wall:

$$\epsilon = \gamma_4 y^3 \tag{4}$$

which leads to the power of 0.33 on Sc-number, as in Eq. 2. Hence, it follows that the Schmidt number exponent in Eq. 1 should be 0.5 for fluid-fluid transport and not 0.44. In what follows we will demonstrate that the data also conform to such a conclusion. To provide support for this viewpoint we analyzed the data for wetted-wall columns (Gilliland and Sherwood, 1934; Barnet and Kobe, 1941). All 215 data points were correlated with the equation of the type:

$$Sh = aRe^b Sc^c \tag{5}$$

using for the exponent c values of 0.5, 0.44 and 0.33. The relative errors are compared in Table 1.

Clearly the relative error of 7.83% for $c = 0.5$ is the lowest. However, even if that were not the case the fact that the errors with $c = 0.44$ and $c = 0.5$ are of the same order and significantly lower than the error when $c = 0.33$ speaks in favor of using the theoretically based exponent for fluid-fluid transport which is $c = 0.5$. The plot for $c = 0.5$ is presented in Figure 1.

Table 1. Coefficients in Eq. 5 and Relative Errors

c	0.5	0.44	0.33
$\Sigma \frac{\lvert y_{cal} - y_{exp} \rvert}{y_{exp}}$	7.57%	8.45%	12.36%
a	0.0318	0.0352	0.0422
b	0.790	0.780	0.759

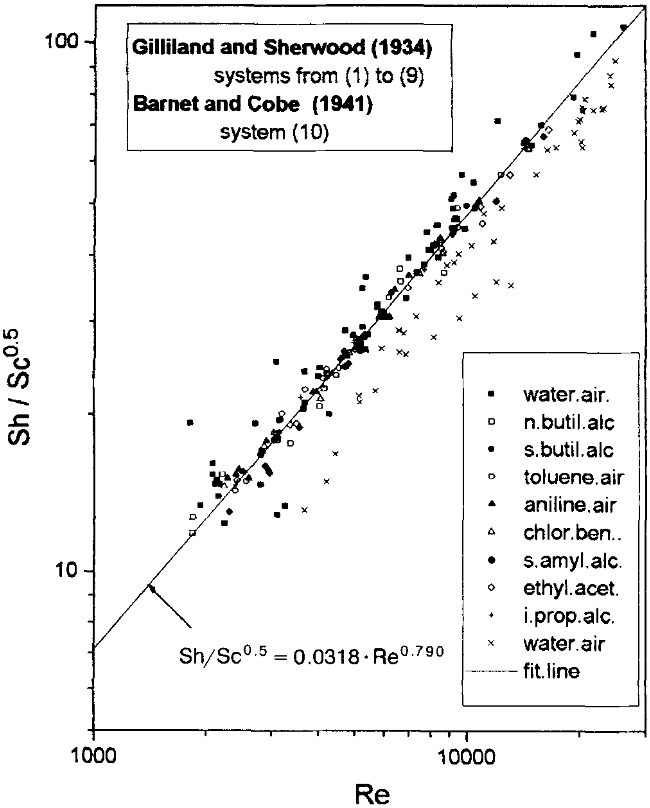

Figure 1. $Sh/Sc^{0.5}$ vs. Re number.

In summary, we have shown that the mass-transfer data taken in wetted wall columns actually represents gas-liquid mass transfer and can be successfully correlated by:

$$\frac{Sh}{Sc^{0.5}} = 0.0318 \cdot Re^{0.790} \tag{6}$$

which has the theoretical square root dependence on the Schmidt number. If prediction of fluid-wall (fluid-solid) transport coefficients is desired, then Eq. 2 contains the proper theoretical 1/3 dependence on the Schmidt number.

Notation

a = coefficient in Eq. 3
b = exponent of Reynolds number in Eq. 3
c = exponent of Schmidt number in Eq. 3
Re = Reynolds number
Sh = Sherwood number
y = distance from the wall
ϵ = eddy viscosity
γ_3, γ_4 = coefficients in Eqs. 3 and 4

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Manuscript received Aug. 4, 1994, and revision received Dec. 6, 1994.